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The Goldie Theorem for H-semiprime algebras ☆

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Abstract

The main result states that, under certain assumptions about a Hopf algebra H, every H-semiprime right Noetherian H-module algebra has a quasi-Frobenius classical right quotient ring. Another question treated in the paper is concerned with the extension of H-module structures to quotient rings. These results have an application to the semiprimeness problem for smash product algebras $A \# H$.

Keywords: The Goldie Theorem; Quotient rings; Hopf algebras; Module algebras; H-semiprime algebras; Smash products

Introduction

Let H be a Hopf algebra over a commutative ring k. An H-module algebra A is called *H-semiprime* if A contains no nonzero nilpotent H-stable ideals. We say that A is *H-simple* if A has no nonzero proper H-stable ideals and A is *H-semisimple* if it is isomorphic to a direct product of finitely many H-simple H-module algebras. This paper is devoted to the proof of the following main result:

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